Project Scheduling Using Fuzzy Logic Approach to Critical Path Analysis

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Article information	Abstract
Key words *Critical path, Project evaluation review technique (PERT), Fuzzy project network.* *Received 26/5/ 2022, Accepted 22/7/ 2022, Available online 6/8/ 2022*	*The aim of this study is to introduce an approach that utilizes a fuzzy logic-based methodology to reduce the impact of uncertainties on the activities of the projects. In this study, the approach presents a method for finding accurate project estimation activities time as compared with the estimation of time of project activities by using the Project Evaluation and Review Technique (PERT). The comparison reveals the proposed method significantly reduced the impact of uncertainties on obtained results to specify the project critical path which led to an improvement in the estimated project completion time.*

I. Introduction

Project management primarily focuses on planning, managing, and organizing the available resources. Some of the activities that should be a part of project management activity are to efficiently guide the project team through all phases and execute the project successfully. Other activities include identifying and efficiently managing the project life cycle and implementing it in the user-centered design process [1].

Matthew J. Liberatore has compared and contrasted the fuzzy logic approach with probability theory, and conclude that fuzzy logic is an alternative approach for modeling uncertainty in project schedule analysis [2].

Chen-Tung Chenat at el, used to study Applying fuzzy method for measuring criticality in project networks and concluded that PERT was the most widely used technique for planning and coordinating large-scale projects. The main assumption in PERT was that the activity durations in a project can be estimated precisely and that they were statistically independent. As a result, PERT appears to lead to poor estimation and inadequate management responses where this assumption does not hold [3].

Luong Duc Long at el, used to study the fuzzy critical chain method for project scheduling under resource constraints and uncertainty, and concluded that the fuzzy critical chain method manage project schedule. By creating a deterministic schedule under resource constraints, and then adding a project buffer at the end of the selected critical chain to cope with uncertainty, the proposed method was practical and useful for scheduling under resource constraints and uncertainty at both planning and execution stages. In the proposed method, the resulting schedule was easily monitored by the project buffer, and it was dynamically revised to provide a better schedule for the remaining activities [4].

Shakeela Sathish at el, studied a simple approach to fuzzy critical path analysis in project networks and concluded that an algorithm to tackle the problem in fuzzy project analysis was proposed. The validity of the proposed method was examined with numerical examples [5].

Igor KREJČÍ at el, used to study that project costs planning in the conditions of uncertainty and concluded that stress the disadvantages of the used approach by Mareš (2000) and the basic problems of the generalization of the CPM formulas to fuzzy numbers. Very wide intervals of the possible values cause by the simple addition of the fuzzy numbers which leads to a decrease in the practical usefulness of the proposed approach. However, the proposed cost analysis was logical in the planning part of the project. In addition, the penalty should be interconnected with the total project duration and not with the single activities. Finally, they determine the possibility of being the cheapest for each project variant, which they feel was the missing part in the original Štikova (2012) [6].

METE MAZLUM, and et al, studied CPM, PERT, and project management with fuzzy logic technique and implementation on a business, and came to the conclusion that CPM and PERT were the two contemporary planning and scheduling techniques that were widely used in construction. Fuzzy and classical implementations of the two methods, which were used in project completion time, were compared. According to the results in general, there were no huge differences between methods [7].

N. Ganapathy Ramasamy at el, a correlated study between time and cost in accordance with fuzzy logic and came to that conclusion scheduling a project is crucial to completing the work as per plan. As a case study, they have taken a small residential apartment. Without scheduling, a project was taken more duration and cost. It has consumed 35% and 24.44% of excess duration and cost to complete the project. Scheduling does make a project finish in a planned approach. LOB scheduling makes the non-repetitive project close soon after than the critical path method. The LOB was more suitable for repetitive projects. Graphical representation of the project was the main benefit in the LOB technique [8].

Md. Mijanoor Rahman at el, used to study those fuzzy numerical results derived from crashing CPM/PERT networks of Padma Bridge in Bangladesh and came to the conclusion that CPM and PERT were widely used in construction, IT, manufacturing, and defense like as contemporary planning and scheduling techniques. These two techniques provide great benefit to the decision makers with being analytical [9].

Farhad Habibi, improved the project time and cost estimation based on PERT using fuzzy logic. Then, concluded that classical methods (CPM) and (PERT) techniques have weaknesses in performance and insufficiency in facing with uncertainties and do not lead to satisfactory results for estimating time and cost of the project. Based on obtain results, a significant reduction in uncertainties impact led to a relative improvement in the estimated time and cost of project completion compared to the CPM and PERT [10].

II. FUZZY CRITICAL PATH ANALYSIS

A critical activity is the one with no float time and should receive special attention (delay in a critical activity will delay the whole project). The critical path is the path(s) through the network that consists of only critical activities.

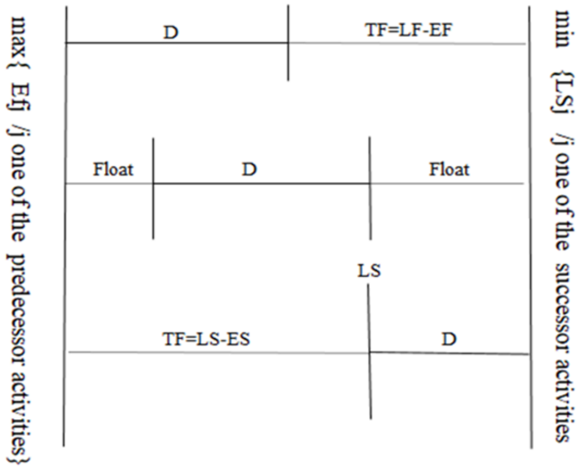

Figure 1. CPM parameters in an activity.

A fuzzy project network is an acyclic digraph, where the vertices represent events, and the directed edges represent the activities, to be performed in a project. We denote this fuzzy project network by $\overline{N} = (\overline{(V)}, \overline{A}, \overline{T})$.

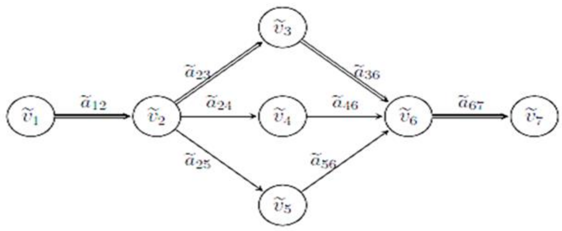

Figure 2. Fuzzy project network 2.

III. FUZZY CRITICAL PATH ANALYSIS

A 5-story building from the start until the end of constructing foundation is taken as a case of the proposed method. The project construction with 170 square meters and consists of 11 activities on an area of is studied.

During the implementation process, the time of each activity was assumed by three experts (N = 3). The activities duration shown in table 1 and obtained results were studied and evaluated using the Fuzzy PERT method.

TABLE 1. PROJECT ACTIVITY

Number	Activity	Duration (days)	prerequisite
1	Equipping the workshop	10	--
2	Excavating	8.75	1
3	Foundation form setting and grading	5	2
4	Lear concrete pouring	2.5	3
5	Purchasing and preparing the rebar	10	1

6	Rebar binding	8.75	4,5
7	Concrete shuttering	6.25	6
8	column base plates design	2.5	7
9	Concrete Pouring	2.5	8
10	Concrete curing	8.75	9
11	Concrete de-shuttering	2.5	10

Project time estimated:

First, using the CPM, the project completion time is obtained equal to 57.5 days, Figure 3 shows the network.

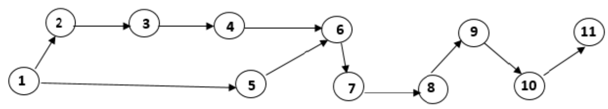

Figure 3. The project network.

TABLE 2. THE OPTIMISTIC DURATION ACTIVITIES, MOST OPTIMISTIC, PESSIMISTIC AND MOST PESSIMISTIC STATES

Activity number	Expert	Most optimistic a_j	Possible optimistic b_j	Possible pessimistic c_j	Most pessimistic d_j	Weight of expert w_{ij}
1	1	7.5	8.75	10	12.5	0.2
	2	6.25	8.75	11.25	13.75	0.5
	3	8.75	10	11.25	12.5	0.3
2	1	6.25	7.5	10	12.5	0.2
	2	7.5	8.75	10	11.25	0.5
	3	6.25	7.5	8.75	10	0.3
3	1	1.25	2.5	3.75	5	0.2
	2	2.5	3.75	5	6.25	0.5
	3	1.25	2.5	3.75	5	0.3
4	1	1.25	2.5	2.5	3.75	0.2
	2	2.5	3.75	3.75	3.75	0.5
	3	1.25	1.25	2.5	2.5	0.3
5	1	6.25	8.75	10	11.25	0.2
	2	8.75	10	12.5	15	0.5
	3	7.5	8.75	10	11.25	0.3
6	1	6.25	7.5	8.75	10	0.2
	2	6.25	8.75	10	11.25	0.5
	3	5	6.25	7.5	10	0.3
7	1	1.25	2.5	2.5	3.75	0.2
	2	2.5	3.75	3.75	5	0.5
	3	2.5	3.75	5	6.25	0.3
8	1	1.25	2.5	3.75	3.75	0.2
	2	2.5	2.5	3.75	3.75	0.5
	3	1.25	2.5	2.5	2.5	0.3
9	1	1.25	2.5	2.5	2.5	0.2
	2	1.25	1.25	1.25	2.5	0.5
	3	1.25	1.25	1.25	2.5	0.3
10	1	6.25	7.5	8.75	10	0.2
	2	7.5	8.75	8.75	10	0.5
	3	6.25	8.75	8.75	10	0.3
11	1	1.25	2.5	2.5	3.75	0.2
	2	1.25	1.25	2.5	2.5	0.5
	3	1.25	2.5	2.5	3.75	0.3

Table 2 shows the durations of activities in the optimistic, most optimistic, pessimistic and most pessimistic states as fuzzy trapezoidal numbers and the weights of 0.2, 0.5 and 0.3 which were all assumed as an expertise opinion.

TABLE 3. CALCULATION OF ACTIVITIES DURATION USING PROPOSED METHOD

Activity number	Most optimistic $\bar{a}_j$	Possible optimistic $\bar{b}_j$	Possible pessimistic $\bar{c}_j$	Most pessimistic $\bar{d}_j$	Defuzzifying	Expected duration T_e	Critical Activity
1	7.25	9.125	11	13.125	10.0625	10.10	√
2	6.875	8.125	9.625	11.125	8.875	8.91	√
3	1.875	3.125	4.375	5.625	3.75	3.75	√
4	1.875	2.75	3.125	3.375	2.9375	2.83	√
5	7.875	9.375	11.25	13.125	10.3125	10.375	
6	5.875	7.75	9	10.625	8.375	8.33	√
7	2.25	3.5	3.875	5.125	3.6875	3.6875	√
8	1.875	2.5	3.375	3.375	2.9375	2.83	√
9	1.25	1.5	1.5	2.5	1.5	1.625	√
10	6.875	8.5	8.75	10	8.625	8.5625	√
11	1.25	1.875	2.5	3.125	2.1875	2.1875	√

Table 2 is shown activities time calculated by proposed method. According to the Table 3 and activities durations defuzzifying, the completion project time was equal to 52.9375 days. The mean of each category, the mean fuzzy number, and the activity processing time were calculated by the following equations:

$$\bar{T}_j^k = (\bar{a}_j^k . \bar{b}_j^k . \bar{c}_j^k . \bar{d}_j^k) = \left(\frac{\sum_{j=1}^N W_{ij}\, a_{ij}^k}{\sum_{j=1}^N W_{ij}} . \frac{\sum_{j=1}^N W_{ij} b_{ij}^k}{\sum_{j=1}^N W_{ij}} . \frac{\sum_{j=1}^N W_{ij} c_{ij}^k}{\sum_{j=1}^N W_{ij}} . \frac{\sum_{j=1}^N W_{ij} d_{ij}^k}{\sum_{j=1}^N W_{ij}}\right) \quad (1)$$

$$DT_j^{(1)} = \frac{\bar{b}_j^k + \bar{c}_j^k}{2} \quad (2)$$

$$\bar{T} = (\bar{a}_2^3, \bar{b}_2^3, \bar{c}_2^3, \bar{d}_2^3) \quad (3)$$

$$\bar{a}_2^3 = \frac{(0.2 * 6.25) + (0.5 * 7.5) + (0.3 * 6.25)}{0.2 + 0.5 + 0.3} = 6.875$$

$$\bar{b}_2^3 = \frac{(0.2 * 7.5) + (0.5 * 8.75) + (0.3 * 7.5)}{0.2 + 0.5 + 0.3} = 8.125$$

$$\bar{c}_2^3 = \frac{(0.2 * 10) + (0.5 * 10) + (0.3 * 8.75)}{0.2 + 0.5 + 0.3} = 9.625$$

$$\bar{d}_2^3 = \frac{(0.2 * 12.5) + (0.5 * 11.25) + (0.3 * 9.10)}{0.2 + 0.5 + 0.3} = 11.125$$

For it's defuzzification, we have:

$$D\bar{T}_2 = \frac{8.125 + 9.625}{2} = 8.875$$

TABLE 4. FORWARD CALCULATION OF THE VALUES OF ES AND EF FOR ACTIVITIES.

Activity	ES	DUR	EF
1	(0,0,0,0)	(7.25, 9.125 , 11 , 13.125)	(7.25, 9.125 , 11 , 13.125)
2	(7.25, 9.125 , 11 , 13.125)	(60875 , 8.125 , 9.625 , 11.125)	(14.125 , 17.25 , 20.625 , 24.25)
3	(14.125 , 17.25 , 20.625 , 24.25)	(4.875 , 3.125 , 4.375 , 5.625)	(16 , 20.375 , 25 , 29.875)
4	(16 , 20.375 , 25 , 29.875)	(1.875 , 2.75 , 3.125 , 3.375)	(17.875 , 23.125 , 28.125 . 33.25)
5	(7.25, 9.125 , 11 , 13.125)	(7.875 , 9.375 , 11.25 13.125)	(15.125 , 18.5 , 22.25 , 26.25)
6	(17.875 , 23.125 , 28.125 . 33.25)	(4.875 , 7.75 , 9 , 10.625)	(23.75 , 30.875 , 37.125 , 43.875)
7	(23.75 , 30.875 , 37.125 , 43.875)	(2.25 , 3.5 , 3.875 , 5.125)	(26 , 34.375 , 41 , 49)
8	(26 , 34.375 , 41 , 49)	(1.875 , 2.5 , 3.375 , 3.375)	(27.875 , 36.875 , 44.375 , 52.375)
9	(27.875 ,36.875 , 44.375 , 52.375)	(1.25 , 1.5 , 1.5 , 2.5)	(29.125 , 38.375 , 45.875 , 54.875)
10	(29.125 , 38.375 , 45.875 , 54.875)	(6.875 , 8.5 , 8.75 , 10)	(36 , 46.875 , 54.625 , 64.875)
11	(36 , 46.875 , 54.625 , 64.875)	(1.25 , 1.875 , 2.5 , 3.125)	(37.25 , 48.75 , 57.125 , 68)

TABLE 5. BACKWARD CALCULATION OF THE VALUES OF LF AND LS FOR ACTIVITIES

Activity	LF	DUR	LS
1	(-17.625 , 2.625 , 17.5 , 38)	(7.25, 9.125, 11, 13.125)	(-30.75, -8.375, 8.375, 30.75)
2	(-6.5 , 12.25 , 25.625 , 44.875)	(6.875, 8.125, 9.625, 11.125)	(-17.625, 2.625, 17.5, 38)
3	(-0.875 , 16.625 , 28.75 , 46.75)	(1.875, 3.125, 4.375, 5.625)	(-6.5, 1.25, 25.625, 44.875)
4	(2.5 , 19.75 , 31.5 , 48.625 ,)	(1.875, 2.75, 3.125, 3.375)	(-0.875, 16.625, 28.75, 46.75)
5	(2.5 , 19.75 , 31.5 , 48.625)	(7.875, 9.375, 11.25, 13.125)	(-10.625, 8.5, 22.125, 40.75)
6	(13.125 , 28.75 , 39.25 , 54.5)	(5.875, 7.75, 9, 10.625)	(2.5, 19.75, 31.5, 48.625)
7	(18.25 , 32.625 , 42.75 , 56.75)	(2.25, 3.5 , 3.875, 5.125)	(13.125, 28.75, 39.25, 54.5)
8	(21.625 , 36 , 45.25 , 58.625)	(1.875, 2.5, 3.375, 3.375)	(18.25, 32.625, 42.75, 56.75)
9	(24.125 , 37.5 , 46.75 , 59.875)	(1.25, 1.5, 1.5, 2.5)	(21.625, 36, 45.25, 58.625)
10	(34.125 , 46.25 , 55.25 , 66.75)	(6.875, 8.5, 8.75, 10)	(24.125, 37.5, 46.75, 59.875)
11	(37.25, 48.75, 57.125,68)	(1.25, 1.875, 2.5, 3.125)	(34.125, 46.25, 55.25, 66.75)

TABLE 6. CALCULATION OF THE VALUES OF TOTAL FLOAT (TF) OF ACTIVITIES

Activity	LF	EF	TF
1	(-17.625,2.625 ,17.5. 38)	(7.25, 9.125, 11, 13.125)	(-30.75, -8.375, 8.375, 30.75)
2	(-6.5, 12.25, 25.625, 44.875)	(14.125, 17.25, 20.625, 24.25)	(-30.75, -8.375, 8.375, 30.75)
3	(-0.875, 16.625, 28.75, 46.75)	(16 ,20.375, 25, 29.875)	(-30.75, -8.375, 8.375, 30.75)
4	(2.5, 19.75, 31.5. 46.625)	(17.875, 23.125, 28.125, 33.25)	(-30.75, -8.375, 8.375, 30.75)
5	(2.5, 19.75, 31.5, 48.625)	(15.125, 18.5, 22.25, 26.25)	(-23.75, -2.5, 13, 33.5)
6	(13.125, 28.75, 39.25, 54.5)	(23.75, 30.875, 37.125, 43.875)	(-30.75, -8.375, 8.375, 30.75)
7	(18.25, 32.625, 42.75, 56.75)	(26, 34.375, 41, 49)	(-30.75, -8.375, 8.375, 30.75)

8	(21.625, 36, 45.25, 58.625)	(27.875, 36.875, 44.375, 52.375)	(-30.75, -8.375, 8.375, 30.75)
9	(24.125, 37.5, 46.75, 59.875)	(29.125, 38.375, 45.875, 54875)	(-30.75, -8.375, 8.375, 30.75)
10	(34.125, 46.25, 55.25, 66.75)	(36, 46.875, 54.625, 46.875)	(-30.75, -8.375, 8.375, 30.75)
11	(37.25, 48.75, 57.125, 68)	(37.25, 48.75, 57.125, 68)	(-30.75, -8.375, 8.375, 30.75)

In network diagrams, there are two possible paths, but the critical path is the one that extends from the start of the project to its end on which all total floats have the zero value.

TABLE 7. CALCULATION OF THE VALUES OF TOTAL FLOAT (TF) OF ACTIVITIES

Activity	TF (fuzzy number)	TF (real number)
1	(-30.75, -8.375, 8.375, 30.75)	0
2	(-30.75, -8.375, 8.375, 30.75)	0
3	(-30.75, -8.375, 8.375, 30.75)	0
4	(-30.75, -8.375, 8.375, 30.75)	0
5	(-23.75, -2.5, 13, 33.5)	5.026
6	(-30.75, -8.375, 8.375, 30.75)	0
7	(-30.75, -8.375, 8.375, 30.75)	0
8	(-30.75, -8.375, 8.375, 30.75)	0
9	(-30.75, -8.375, 8.375, 30.75)	0
10	(-30.75, -8.375, 8.375, 30.75)	0
11	(-30.75, -8.375, 8.375, 30.75)	0

IV. Results of time estimation

Supposed experts' opinion through and the proposed method, the completion time was 52.9375 days compared to 57.5 and 58.5375 days through the CPM and PERT respectively. It is clear that the improvement of project completion time estimation was achieved. That means, the uncertainties affecting is extremely reduced on the results and scheduling was closer to reality.

V. CONCLUSION

Traditional methods such as CPM have poor performance estimated of project time which leads to unsatisfactory and inefficiency resulting in facing uncertainties.

To deal with such problems, experienced experts help, and fuzzy theory is considered the basic way to deal with such problems. For this reason and to reduce the impact of uncertainties on the estimating time, the present project provided a fuzzy-based methodology was introduced.

Assumed expert's activities time (most pessimistic, possible pessimistic, most optimistic, and possible optimistic) are expressed as fuzzy numbers in this methodology.

The estimation was carried out after defuzzifying. In order to understand how this framework was performed and evaluate the performance of the proposed framework, a case study was executed about a construction project.

Depending on the compared results, the impact of uncertainties was a lack of efficiency on obtained results compared to the CPM.

Finally, the results show there is a relative improvement in the estimated time by using the proposed method.

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