Mathematical Applications Of a Rough Topology In Medical Events (COVID19)

Faraj.A.Abdunbi, Ahmed.shletie,
College of Science, Ajdabyia University

Article information

Key words
Rough Sets, Lower Approximation, Upper Approximation Rough Topology, Core

Received 26 February 2021, Accepted 22 March 2021, Available online 01 April 2021

Abstract

This study aims to show that we can use the modern mathematics such as a rough topology to analyze many real life problems. The rough set theory is a good formal tool for processing incomplete information in an information system. The concepts of a rough topology based on the concepts of upper and lower approximations of a given set called universal. Rough set theory use in many areas. Here, we will apply this theory in a new medical event (COVID19). Our work us the rough topology to analyze many real life problems. Our result we try to find the good deciding factor for the diseases COVID19..

I. INTRODUCTION

The theory of rough set theory have shown by Pawlak in 1982[1]. This theory is a good tool for modeling in information. It is a deal with the concepts by approximations of sets and considered as one of the first non-statistical approaches in data analysis. In recently 40 years, several interesting applications of the theory have come up and many researching develop and use rough theory in many areas such as Artificial Intelligence [3], Cognitive Sciences, Data Mining [2], and life sciences. In [4], Pooja and others consider the concept of rough set theory and its applications in decision-making Processes. In [5], Pattaraintakorn and Cercone, present a short survey of ongoing research and a case study on integrating rough set theory and medical application. The rough set theory has advantage in data analysis that does not require any preliminary or additional information of the data. Rough set theory different from fuzzy sets since the rough sets have precise boundaries whereas fuzzy set theory is generally based on ill-defined sets of data, where the bounds are not precise and hence fuzzy predictions tend to deviate from exact values. The concepts of rough set based on the lower and upper approximations of a set are analogous to the interior and closure operations in a topology generated by data. In [6], El Sayed and others introduce a method to make a topological reduction of the attributes generated from a multi-information table. In this paper, we have introduced a new concept is called topology rough topology. In addition, we applied the concept of topological basis to find the deciding factors for COVID19.

II. 2. PRELIMINARIES

Suppose that U (universe) be a non-empty finite set of objects. Suppose that R an equivalence relation on U. The pair (U,R) is called the approximation space. We use U/R to denote the family of all equivalent class of R determined by x denoted by $([x]_R)$.
Let X be a subset of U we write X^C to denote the complement of X in U.

Definition 2-1[1]: The lower approximation of X with respect to R is the set of all objects is denoted by $R^{(X)} = \{[x]_R : [x]_R \subseteq X\}$. And the upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $R_{(X)} = \{[x]_R : [x]_R \cap X \neq \varphi\}$.
Definition 2-2[1]:For an approximations space (U,R), the boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by BRX= $R^{(X)} - R_{(X)}$.
Note that, the set X is said to be rough with respect to R if BRX $\neq \varphi$.

Proposition 2.3[1]: If (U, R) is an approximation space and X and Y are subsets of U, then:

i) $R^{(X)} \subseteq R_{(X)}$.

ii) $R^{(\varphi)} \subseteq R_{(\varphi)} = \varphi, R^{(U)} \subseteq R_{(U)} = U$. φ

iii) $R^{(X\cup Y)} = R^{(X)} \cup R^{(Y)}$.
iv) $R_{(X\cup Y)} \supseteq R_{(X)} \cup R_{(Y)}$.
v) $R_{(X\cap Y)} = R_{(X)} \cap R_{(Y)}$.
vi) $R^{(X\cap Y)} \subseteq R^{(X)} \cap R^{(Y)}$.
$R^{(X)} \subseteq R^{(Y)}, R_{(X)} \subseteq R_{(X)}$ *whenever* $X \subseteq Y$

vii) $R_{(X^C)} = [R_{(X)}]^C$ *amd* $R^{(X^C)} = [R_{(X)}]^C$.
viii) $R_{(R(X))} = R^{R(X)} = R^{(X)}$.
ix) $R^{R(X)} = R_{R(X)} = R^{(X)}$.

III. 3. Rough Topology

Suppose that U is the universe of objects and R is an equivalence relation on U. We use the concept of a topology called rough topology in terms of the lower and upper approximations.

Definition 3-1[3]: For $X \subseteq U$, we define the rough topology by $\tau_R = \{U, \varphi, R^{(X)}, R_{(X)}, BRX\}$.

Note that, we have:
1) U and $\varphi \in \tau_R$.
2) Since $R_{(X)} \subseteq R^{(X)}$, $R_{(X)} \cup R^{(X)} = R^{(X)} \in \tau_R$. Also, $R^{(X)} \cup BR(X) = R_{(X)} \in \tau_R$ and $R_{(X)} \cup BR(X) = R^{(X)} \in \tau_R$. Also, $R^{(X)} \cap R_{(X)} = R^{(X)} \in \tau_R$; $R^{(X)} \cap BR(X) =$ BR(X) $\in \tau_R$ and $R_{(X)} \cap$BR(X)$= \varphi \in \tau_R$.

Proposition 3.2[3]: Suppose that U be the universe. Let R be an equivalence relation on U and $\tau_R = \{U, \varphi, R^{(X)}, R_{(X)}, BR(X)\}$ where $X \subseteq U$. then τ_R
Is a topology on U called the rough topology on U with respect to X.

Proof: it clear from definition 3-1 τ_R satisfies the following axioms:
1) U *and* $\varphi \in \tau_R$.
2) $\cup$(the elements of any sub collection of τ)$\in \tau_R$.
3) $\cap$(the elements of any finite sub collection of τ_R)$\in \tau_R$.

Note that, τ_R (U, τ_R, X) is called the rough topological space.

Example 3.3: Suppose that U =\{*red, green, black, yellow, pink*\}. Let U/R=\{\{*red, green*\},\{*black ,yellow*\},\{*pink*\}\}, the family of equivalence classes of U by the equivalence relation R and X = \{*red, black, yellow*\}. Then $R^{(X)}$ = \{*red, green, black, yellow*\}, $R_{(X)}$ = \{*black, yellow*\} and $BR(X)$=\{*red, green* \}. Therefore the rough topology τ_R = \{*U, φ*, \{*red, green, black, yellow*\}, \{*black, yellow*\}.

Remake 3-4: suppose that If τ_R is the rough topology on U with respect to X, then the set $B = \{U, R_{(X)}, BR(X)\}$ is the basis for τ_R.

IV. 4. Rough Topology in Covid 19

In this section, we try to apply the concepts of rough topology in the new disease (COVID19). There have been recent cases of suspicion and infection, and there are symptoms associated with severe disease. It causes coughing, headache, fever, and runny nose. Symptoms are similar to the common cold, as is COVID19. Some symptoms are not usually life threatening. However, the cough can persist and may cause death if the period is relatively long. Usually the patient acquires immunity from the disease from the infection, and therefore infection is very rare for a period that may exceed six months. The disease has spread throughout the world, especially in Libya. Consider table 1, which gives data for eight patients.

Table 1

(P)	(T)	H	R	K	Covid19
P1	High	Y	Y	Y	Y
P2	High	N	N	Y	N
P3	High	N	N	Y	Y
P4	V.High	Y	N	N	N
P5	High	Y	Y	N	N
P6	V.High	Y	N	Y	Y
P7	Normal	Y	N	Y	N
P8	V.High	Y	N	Y	Y

We can see, the rows represent the objects (the patients) and the columns of the table represent the attributes (the symptoms for Coivd19) and the entries in the table are the attribute values.

The patient P_1 is characterized by the value set (*Temperature, High*), (*Headache, Yes*), *(Runny nose, Yes*), *Cove, Yes*) and (*Coivd19, Yes*), which gives information about the patient P_1.
Also, we can see in the table, the patients P_1, P_4, P_5, P_6, P_7 *and* P_8 are indiscernible with respect to the attribute '*Head ache*'.
The attribute 'head ache ' generates two equivalence classes$\{P_1, P_4, P_5, P_6, P_7, P_8\}$ and $\{P_2, P_3\}$.
If the attributes '*Cove*' and '*Headache*' generate the equivalence classes $\{P_1,P_6,P_7,P_8\}$, $\{P_2,P_3\}$,$\{P_4,P_5\}$. The equivalence classes for the attributes Cove, headache, runny nose and Temperature are $\{P_1\}$, $\{P_2,P_3\}$,$\{P_4\}$,$\{P_5\}$,$\{P_6,P_8\}$ and $\{P_7\}$.
We, have from definition 2-1, the lower approximation $R^{(X)} = \{[x]_R : [x]_R \subseteq X\}$.= $\{P_1,P_6,P_8\}$ and the upper approximation $R_{(X)} = \{P_1, P_2, P_3, P_6, P_8\}$ and hence the boundary region BRX= $R^{(X)} - R_{(X)}$ = $\{P_2, P_3\} \neq \emptyset$. It rough set.
Note that, the patients P_2 and P_3 can not be uniquely classified in view of the available knowledge.

However, he patients P_1, P_6 and P_8 display symptoms, which enable us to classify them with certainty as having Coivd19.

We can look not necessary to look at all condition attributes in an information system make clear decision attribute before decision rules are generated.

Remake 4-1. Sometimes, It may happen that the decision attribute depends not on the whole set of condition attributes but on a subset of it . In addition, we are interested to find the subset that given by the core.

Now, We suppose that $U = \{P_1,P_2,...,P_8\}$. Let $X = \{P_1,P_3,P_6,P_8\}$, the set of patients having Coivd19.

We assume R be the equivalence relation on U with respect to the condition attributes.

We have the family of equivalence classes corresponding to R is given by $U/I(R) = \{\{P_1\}, \{P_2,P_3\}, \{P_4\}, \{P_5\}, \{P_6,P_8\}, \{P_7\}\}$.

So, The lower approximation of X with respect to R is $R^{(X)}=\{P_1,P_6,P_8\}$ and the upper approximation $R_{(X)} = \{P_1,P_2,P_3,P_6,P_8\}$. Therefore, the rough topology on U with respect to X is $\tau_R = \{U, \varphi, \{P_1,P_6,P_8\}, \{P_1,P_2,P_3,P_6,P_8\}, \{P_2,P_3\}\}$. And, the basis for this topology τ_R is $\beta R = \{U, \{P_1,P_6,P_8\},\{P_2,P_3\}\}$.

Now, if we omitting the attribute 'cove' from the set of condition attributes, then we get the family of equivalence classes corresponding to the resulting set of attributes is $U/I(R\text{-}(K)) = \{\{P_1,P_5\}, \{P_2,P_3\}, \{P_4\}, \{P_6,P_8\}, \{P_7\}\}$. Moreover, the lower approximations is $(R\text{-}(K))^{(X)}= \{P_6,P_8\}$ and the upper approximations is $(R\text{-}(K))_{(X)}= \{P_1,P_2,P_3,P_5,P_6,P_8\}$. Hence, $\tau_R\text{-}(K) =\{U,\varphi, \{P_6,P_8\}, \{P_1,P_2,P_3,P_5,P_6,P_8\}, \{P_1,P_2,P_3,P_5\}\}$.

In addition, the basis $\beta R\text{-}(K)=\{U,\{P_6,P_8\}, \{P_1,P_2,P_3,P_5\}\}\neq \beta R$.

Bui, if we omitting other attribute such as 'Headache from the set of condition attributes, we get the family of equivalence classes corresponding to the resulting set of attributes is $U/I(R\text{-}(H))=\{\{P_1\},\{P_2,P_3\}, \{P_4\},\{P_5\}, \{P_6,P_8\},\{P_7\}\}$ which is the same as $U/I(R)$ and hence $\tau_R\text{-}(H) = \tau_R$ and $\beta R\text{-}(H) = \beta R$.

On removal of the attribute 'Runny nose', we get $U/I(R\text{-}(R))=\{\{P_1\},\{P_2,P_3\},\{P_4\},\{P_5\},\{P_6,P_8\}, \{P_7\}\}= U/I(R)$ and hence $\tau_R\text{-}(R) = \tau_R$ and $\beta R\text{-}(R) = \beta R$. When the attribute 'Temperature' is omitted, $U/I(R\text{-}(T)) = \{\{P_1\}, \{P_2,P_3\}, \{P_4\}, \{P_5\}, \{P_6,P_7,P_8\}\}$.

Then the lower approximation $(R\text{-}(T))\ (X)=\{P_1\}$; and the Uppe approximation $(R\text{-}(T))(X)= \{P_1,P_2,P_3,P_6,P_7,P_8\}$. Moreover, $,\tau(R\text{-}(T))=\{U ,\varphi, \{P_1\}, \{P_1,P_2,P_3,P_6,P_7,P_8\},\{P_2,P_3,P_6,P_7,P_8\}\}$.

And the basis $\beta R\text{-}(T)=\{U,\{P_1\},\{P_2,P_3,P_6,P_7,P_8\}\}\neq\beta R$.

Suppose that $M =\{K,T\}$, then the basis for the rough topology corresponding to M is $\beta M =\{U,\{P_6,P_8\}, \{P_1,P_2,P_3\}\}$. Moreover, we have $\beta M \neq \beta R\text{-}(x)$ for all x in M. Therefore, $CORE(R) = \{K,T\}$.

Now, if we Suppose that $X =\{P_2,P_4,P_5,P_7\}$,the set of patients not having Coivd19. We can see, $U/I(R) = \{\{P_1\}, \{P_2,P_3\}, \{P_4\}, \{P_5\}, \{P_6,P_8\}, \{P_7\}\}$.

Note that, the lower approximation is $R^{(X)}= \{P_4,P_5,P_7\}$.

And the upper apportion is $R_{(X)} = \{P_2,P_3,P_4, P_5,P_7\}$. Therefore $\tau_R = \{U, \varphi, \{P_4,P_5,P_7\}, \{P_2,P_3,P_4,P_5,P_7\}, \{P_2,P_3\}\}$ and the $\beta R = \{U, \{P_4,P_5,P_7\},\{P_2,P_3\}\}$.

If we remove the attribute 'Cove', then we get $U/I(R\text{-}(K)) = \{\{P_1,P_5\}, \{P_2,P_3\}, \{P_4\}, \{P_6,P_8\}, \{P_7\}\}$.

And the lower approximation is $(R\text{-}(K))^{(X)} = \{P_4,P_7\}$; and the upper approximation is $(R\text{-}(k))_{(X)} = \{P_1,P_2,P_3,P_4,P_5,P_7\}$. Hence $\tau_R\text{-}(K) = \{U, \varphi, \{P_4,P_7\}, \{P_1,P_2,P_3,P_4,P_5,P_7\}, \{P_1,P_2,P_3,P_5\}\}$. However, the basis $\beta R\text{-}(K) =\{U,\{P_4,P_7\}, \{P_1,P_2,P_3,P_5\}\} \neq \beta R$.

Remark 4-2: If we remove the attribute 'Headache', then, $U/I(R\text{-}(H)) = \{\{P_1\},\{P2,P3\}, \{P_4\}, \{P_5\},\{P_6,P_8\}, \{P_7\}\}$ which is the same as $U/I(R)$ and hence $\beta R\text{-}(H) = \beta R$.

If we remove of the attribute 'Runny nose', then we get$U/I(R(R))=\{\{P_1\},\{P_2,P_3\},\{P_4\},\{P_5\},\{P_6,P_8\},\{P_7\}\}$,which is the same as $U/I(R)$ Hence $\tau_R\text{-}(R) = \tau_R$ and $\beta R\text{-}(R) = \beta R$.

If we omitting the attribute 'Temperature', then $U/I(R\text{-}(T)) = \{\{P_1\}, \{P_2,P_3\}, \{P_4\}, \{P_5\}, \{P_6,P_7,P_8\}\}$.

Also, the lower approximation $(R\text{-}(T))^{(X)} = \{P_4,P_5\}$; and the upper approximation is $(R\text{-}(T))_{(X)} = \{P_2,P_3,P_4,P_5,P_6,P_7, P_8\}$. Therefore, $\tau_R\text{-}(T) = \{U, \varphi, \{P_4,P_5\}, \{P_2, P_3,P_4,P_5,P_6,P_7,P_8\}, \{P_2,P_3,P_6,P_7,P_8\}\}$. Moreover, the basis $\beta R\text{-}(T)=\{U,\{P_4,P_5\}, \{P_2,P_3,P_6,P_7,P_8\}\} \neq \beta R$.

If we suppose that $M = \{K,T\}$,then we get $U/I([s])= \{\{P_1,P_2,P_3\}, \{P_4,P_5\}, \{P_6,P_8\}, \{P_7\}\}$, the lower approximation is $[s]^{(X)}= \{P_4,P_5,P_7\}$ and the upper approximation is $[s]_{(X)}= \{P1, P2, P3, P4, P_5, P_7\}$ where $[s]$ is the equivalence relation on I with respect to I. Therefore, $\beta M = \{U, \{P_4,P_5,P_7\},\{P_1,P_2,P_3\}\} \neq \beta R\text{-}(x)$ for every x in M. We can see that $CORE(R)= \{K,T\}$. It noted from our result that we conclude that "*cove*" and "*Temperature*" are the main and necessary traits for deciding whether a patient suffers from Coivd19.

Conclusions

Our result showed us we could use the mathematics such as a rough topology to analyze many real life problems. We showed that real-world problems can be dealt with an approximate topology. We apply the concept of basis to make find the good decision about the Virus disease 'Covid19'. In particular, it has been reported in Libya, and specifically in Benghazi. It noted that symptoms such as temperature and Cove are the determining factors for identifying Coivd19. However, it noted that from a clinical point of view, our results define an approximate topological model with what medical experts recommend regarding Covid19 disease. It should be note here, and through many studies, and this research, that we can use and apply the concept of approximate topology proposed for more general and complex types and information systems in the future.

Acknowledgement. I would like to thank Mr. Mari Al-Doukali who the member of the teaching staff of the Department of medical laboratories at the Higher

Institute of Science and technology, Saluq to collect data from patients.

REFERENCES

[1] [1] Pawlak, Z., & Skowron, (2007) A. Rudiments of rough sets. Information sciences, 177(1), , pp. 3-27.

[2] [2] Mohammad Hossein, ((2008))Application of Rough Set Theory in Data Mining for Decision Support Systems (DSSs), Journal of Industrial Engineering 125 – 34.

[3] [3] G.Y. Wang and Z. Zheng, Y. Zhang, ((2002),)RIDAS- a rough set based intelligent data analysis system,International Conference on Machine Learning & Cybernetics pp. 646-649

[4] [4] Pooja and others, Concept of Rough Set Theory and its Applications in Decision Making Processes, International Conference on Advances in Computational Techniques and Research Vol. 6, Special Issue 2, 2017.

[5] [5] P. Pattaraintakorn, N. Cercone,((2008))Integrating rough set theory and medical applications, Applied Mathematics Letters 21400–403.

[6] [6] M. El Sayed1, el,(2021) Topological approach for decision-making of COVID-19 infection via a nano-topology model, AIMS Mathematics, 6(7): 7872–7894.