Fuzzy Rough Network Problems

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Abstract

In conventional network problems, it is assumed that decision maker is certain about the parameters (distance, time, …) between different nodes. But in real life situations, there always exist uncertainty about the parameters between different nodes. In this paper, we present algorithm for solving the fuzzy rough network problem, in this problem all parameters between different nodes are presented by triangular fuzzy rough numbers. By using the proposed algorithm a decision maker can obtain the optimal shortest path and optimal fuzzy rough shortest distance between source node and destination node. To illustrate algorithm a numerical example is solved and the obtained results are discussed

I. Introduction

The shortest path problem concentrates on finding the path with minimum distance. The shortest path problem has received lots of attention from researchers in the past decades, because it is important to many applications such as communication, transportation, scheduling and routing. Klein (1991), presented new models based on fuzzy shortest paths and also given a general algorithm based on dynamic programming to solve the new models. In a network, the arc length may represent time or cost. Conventionally, it is assumed to the crisp. However, it is difficult for decision makers to specify the arc distance. For example, using the same modem to transmit the data from node a to node b in a network, the data transmission time may not be the same every time. Therefore, in real world, the arc distance could be uncertain. Fuzzy set theory, as presented by Zadeh [1], is frequently utilized to deal with uncertainty, and using membership function to describe uncertainties. Takahashi and Yama kami [2] discussed the shortest path problem with fuzzy parameters. Nayeem and Pal [3] considered a network with its arc lenggths as imprecise number, instead of a real number, namely, interval number and triangular fuzzy number. Tajdin et al [4] proposed a new approach and an algorithm to find a shortest path in mixed network having various fuzzy arc lengths. Amit and Manjot [5] presented an algorithm for solving network flow problems with fuzzy arc lengths. Ravi Shankar et al [6] used a new defuzzification formula for fuzzy number and applied to the float time (slack time) for each activity in the fuzzy project network to find the critical path. Yakhchali and Ghodsypour [7] introduced the problems of determining possible values of earliest and latest starting times of an activity in networks with minimal time lags and imprecise durations that are represented by means of interval or fuzzy numbers.

Here, we present algorithm to find a shortest path and fuzzy rough shortest distance in network problem. In this problem the network having various fuzzy rough arc distance, the remainder of the paper is organized as follows: In section 2, some basic definition, arithmetic operations are reviewed. In section 3, the formulation of fuzzy rough shortest path problem presented, algorithm for solving it's, to illustrate algorithm numerical example is solved. The conclusions are discussed in section 4.

II. Preliminaries

A. Fuzzy Rough interval

In this section, the definition of rough interval, fuzzy rough interval, fuzzy rough number and basic operations for triangular fuzzy rough numbers are given. For more details see [8 -11].

Definition 2.1. The qualitative value X is called a rough interval when one can assign two closed intervals X^{LAI} and X^{UAI} on real set $\mathcal{R}$ to it where $X^{LAI} \subseteq X^{UAI}$. Moreover,

i. If $x \in X^{LAI}$ then X surely takes x (denoted by $x \in X$).

ii. If $x \in X^{UAI}$ then X possibly takes.

iii. If $x \notin X^{UAI}$ then X surely dose not takes x (denoted by $x \notin X$). X^{LAI} and X^{UAI} are called the lower approximation interval (LAI) and upper approximation interval (UAI) of X, respectively. Further, X is denoted by $X^R = [\ X^{LAI} : X^{UAI}\ ]$.

Definition 2.2. Let X be denoted a compact set of real numbers, a fuzzy rough interval $\tilde{X}^R$ is defined as $\tilde{X}^R = [\ \tilde{X}^{LAI} : \tilde{X}^{UAI}\ ]$ where $\tilde{X}^{LAI}$ and $\tilde{X}^{UAI}$ are fuzzy set called lower and upper approximation fuzzy number of $\tilde{X}^R$ with $\tilde{X}^{LAI} \subseteq \tilde{X}^{UAI}$.

Definition 2.3. A fuzzy rough numbers $\tilde{A}^R$ is a convex normalized fuzzy rough interval of the real line $\mathcal{R}$ whose membership function is piecewise continuous.

Definition 2.4. A fuzzy rough number $\tilde{A}^R$ is a triangular fuzzy rough number denoted by $\tilde{A}^R = [(a^{LL}, a^M, a^{UL}) : (a^{LU}, a^M, a^{UU})]$ where $a^{LL}, a^M, a^{UL}, a^{LU}$ and $a^{UU} \in \mathcal{R}$ such that $a^{LU} \leq a^{LL} \leq a^M \leq a^{UL} \leq a^{UU}$ and the membership functions defined by:

$$\tilde{A}^R = \begin{cases} \mu_{\tilde{A}^L}(x) = \begin{cases} \frac{x-a^{LL}}{a^M-a^{LL}} & a^{LL} \leq x \leq a^M \\ , & \\ \frac{x-a^{UL}}{a^M-a^{UL}} & a^M \leq x \leq a^{UL} \\ , & \\ 0 & otherwise \end{cases} \\ \mu_{\tilde{A}^U}(x) = \begin{cases} \frac{x-a^{LU}}{a^M-a^{LU}} & a^{LU} \leq x \leq a^M \\ , & \\ \frac{x-a^{UU}}{a^M-a^{UU}} & a^M \leq x \leq a^{UU} \\ , & \\ 0 & otherwise \end{cases} \end{cases}$$

Note that $\tilde{A}^L = (a^{LL}, a^M, a^{UL})$, $\tilde{A}^U = (a^{LU}, a^M, a^{UU})$ and $\tilde{A}^L \subseteqq \tilde{A}^U$.

Example 1 : A triangular fuzzy rough numbers define as: $\tilde{A}^R = [\ (4,5,6) : (2,5,7)\ ]$, we can be defined the membership functions $as:$

$$\tilde{A}^R = \begin{cases} \mu_{A^L}(x) = \begin{cases} x-4 & 4 \leq x \leq 5 \\ & , \\ 6-x & 5 \leq x \leq 6 \\ & , \\ 0 & otherwise \end{cases} \\ \mu_{A^U}(x) = \begin{cases} \frac{x-2}{3} & 2 \leq x \leq 5 \\ & , \\ \frac{x-7}{-2} & 5 \leq x \leq 7 \\ & , \\ 0 & otherwise \end{cases} \end{cases}$$

the membership function of $\tilde{A}^R = [\ (4,5,6) : (2,5,7)\ ]$ is shown in figure 1.

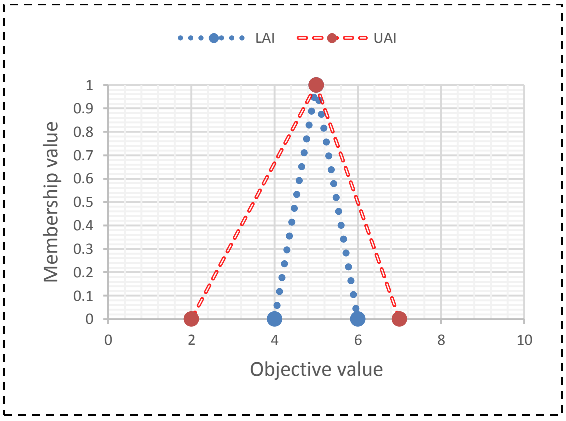

Figure 1. the membership functions for Example 1.

B. *Basic Operation For Triangular Fuzzy Rough Number*

Let $\tilde{A}^R = [(a^{LL}, a^M, a^{UL}) : (a^{LU}, a^M, a^{UU})]$ and $\tilde{B}^R = [(b^{LL}, b^M, b^{UL}) : (b^{LU}, b^M, b^{UU})]$ be two triangular fuzzy rough numbers, where $\tilde{A}^R and\ \tilde{B}^R \geq 0$ then the arithmetic operations are defined by:

1. **Addition:**

$\tilde{A}^R + \tilde{B}^R \cong^R [\ \tilde{A}^L + \tilde{B}^L : \tilde{A}^U + \tilde{B}^U\ ]$ Where

$$\begin{cases} \tilde{A}^L + \tilde{B}^L \cong (\ a^{LL} + b^{LL},\ a^M + b^M,\ a^{UL} + b^{UL}), \\ \tilde{A}^U + \tilde{B}^U \cong (a^{LU} + b^{LU},\ a^M + b^M,\ a^{UU} + b^{UU}) \end{cases}$$

2. **Subtraction:**

$$\tilde{A}^R - \tilde{B}^R \cong^R [\tilde{A}^L - \tilde{B}^L : \tilde{A}^L - \tilde{B}^L\ ]$$

$$where \begin{cases} \tilde{A}^L - \tilde{B}^L \cong (\ a^{LL} - b^{UL},\ a^M - b^M,\ a^{UL} - b^{LL}) \\ , \\ \tilde{A}^U - \tilde{B}^U \cong (\ a^{LU} - b^{UU},\ a^M - b^M,\ a^{UU} - b^{LU}) \end{cases}$$

3. **Multiplication:**

$$\tilde{A}^R \times \tilde{B}^R \cong^R [\tilde{A}^L \times \tilde{B}^L : \tilde{A}^U \times \tilde{B}^U]$$

$$where \begin{cases} \tilde{A}^L \times \tilde{B}^L \cong (\ a^{LL} \times b^{LL},\ a^M \times b^M,\ a^{UL} \times b^{UL}), \\ \tilde{A}^U \times \tilde{B}^U \cong (\ a^{LU} \times b^{LU},\ a^M \times b^M,\ a^{UU} \times b^{UU}) \end{cases}$$

4. **Division:**

$$if \quad \widetilde{B}^R \neq \widetilde{0}^R \quad then$$
$$\widetilde{A}^R \div \widetilde{B}^R \ \cong^R [\ \widetilde{A}^L \div \widetilde{B}^L \ : \ \widetilde{A}^U \div \widetilde{B}^U\ ]$$

$$where \begin{cases} \widetilde{A}^L \div \widetilde{B}^L \cong (\ a^{LL} \div b^{UL}, a^M \div b^M\ , a^{UL} \div\ b^{LL}\ ), \\ \widetilde{A}^U \div \widetilde{B}^U \cong (\ a^{LU} \div b^{UU}, a^M \div b^M\ , a^{UU} \div b^{LU}\ ) \end{cases}$$

C. Ranking Function for triangular Fuzzy Rough Numbers [10,12]

A ranking function is a function $\Re : \tilde{F}^R \longrightarrow \mathcal{R}$, where $\widetilde{F}^R$ is a set of all triangular fuzzy rough numbers defined on set of real numbers, which maps each triangular fuzzy rough number into the real number. The ranking function for a triangular fuzzy rough $\widetilde{A}^R = [(a^{LL}, a^M, a^{UL}) : (a^{LU}, a^M, a^{UU})]$ denoted by $\Re(\widetilde{A}^R)$ can be defined as:

$$\Re(\widetilde{A}^R) = \frac{1}{8}(a^{LL} + a^{UL} + 4a^M + a^{LU} + a^{UU}).$$

Let $\widetilde{A}^R$ *and* $\widetilde{B}^R$ be two triangular fuzzy rough numbers, then

i. $\widetilde{A}^R \precsim^R \widetilde{B}^R \ \boldsymbol{iff}\ \Re(\widetilde{A}^R) < \Re(\widetilde{B}^R)$

ii. $\widetilde{A}^R \succsim^R \widetilde{B}^R \ \boldsymbol{iff}\ \Re(\widetilde{A}^R) > \Re(\widetilde{B}^R)$

iii. $\widetilde{A}^R \cong^R \widetilde{B}^R \ \boldsymbol{iff}\ \Re(\widetilde{A}^R) = \Re(\widetilde{B}^R)$

III. FUZZY ROUGH SHORTEST PATH PROBLEMS

In this section we present new models for a network where the arc lengths between deferent nodes are triangular fuzzy rough numbers. The fuzzy rough shortest path problem, determines the shortest path and fuzzy rough shortest distance between a source node and destination node in a network. The notations that will be used in the fuzzy rough shortest path problem are as follows:

(N, A): where N is the set of nodes and A is the set of arcs.

Nd (j) : The set of all predecessor nodes of node j.

$\widetilde{d}_i^R$: The fuzzy rough distance between node i and first (source) node.

$\widetilde{d}_{ij}^R$: The fuzzy rough distance between node i and node j.

Remark: Let $\widetilde{A}_i^R$; $i = 1,2, \dots .n$ be a set of triangular fuzzy rough numbers, If $\Re(\tilde{A}_k^R) < \Re(\tilde{A}_i^R)$ for all i, then the triangular fuzzy rough number $\widetilde{A}_k^R$ is the minimum of $\widetilde{A}_i^R$.

A. *Algorithm for computing a shortest path.*

A new algorithm is presented for finding the optimal shortest path and optimal fuzzy rough shortest distance between source node (say node 1) and destination node (say node n). The steps of the algorithm are as follows:

Step 1. Suppose $\widetilde{d}_1^R = \widetilde{0}^R = [(0,0,0) : (0,0,0)]$ and label the source node (say node 1) as $\{\widetilde{0}^R, -\}$.

Step 2. Find $\widetilde{d}_j^R = \boldsymbol{minimum}\left\{\ \widetilde{d}_i^R + \widetilde{d}_{ij}^R \ : i \in Nd\,(j)\,,\ j = 2,3, \dots, n\right\}$.

Step 3. If minimum occurs corresponding to unique value of i ($i.e\ \ i = m$) then label node j as $\{\ \tilde{d}_j^R, m\ \}$, If minimum occurs corresponding to more than one value of i then it represents that there are more than one fuzzy rough distance between source node i and node j, but fuzzy rough distance along all paths is $\widetilde{d}_j^R$, so choose value of i (using step 2).

Step 4. Let the destination node (node n) be labeled as $\left\{\ \widetilde{d}_n^R\ ;\ r\ \right\}$, $r = 2,3, \dots, n-1$, then the optimal fuzzy rough shortest distance between source node and destination node is $\widetilde{d}_n^R$.

Step 5. Since destination node is labeled as $\{\ \tilde{d}_n^R\ ;\ r\}$ so to find the optimal shortest path between source node and destination node, check the label of destination node n. Since the $\{\ \tilde{d}_n^R\ ;\ r\}$, which represents that we are coming from node r, now check the label of node r, and so on. Repeat the same procedure until node 1 is obtained. By the combining all the nodes then the optimal shortest path can be obtained. To illustrate the algorithm the numerical example presented.

Example 2:

The problem is to find the optimal shortest path between source node (say node 1) and destination node (say node 7) on the network Figure 2, where the triangular fuzzy rough arc distance are presented in Table 1.

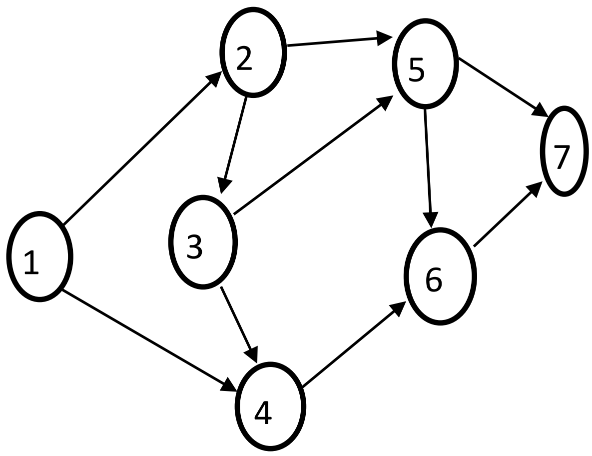

Figure 2. The network of Example 2

Table 1. The arc distance for example 2

Arc	Distance	Arc	Distance
(1,2)	$[(9,12,15):(6,12,18)]$	(3,5)	$[(7,14,17):(5,14,19)]$
(1,4)	$[(20,25,30):(13,25,33)]$	(4,6)	$[(20,25,28):(17,25,33)]$
(2,3)	$[(12,16,21):(7,16,25)]$	(5,6)	$[(8,9,10):(5,9,13)]$
(2,5)	$[(6,11,16):(2,11,20)]$	(5,7)	$[(18,21,26):(15,21,31)]$
(3,4)	$[(15,20,24):(12,20,27)]$	(6,7)	$[(8,11,13):(4,11,14)]$

Solution: since node 7 is the destination, so $n=7$
Assume $\tilde{d}_1^R=[(0,0,0):(0,0,0)]$ and label the source node (say node 1) as $\{[(0,0,0):(0,0,0)]\ ;\ -\}$

Now the values of $\tilde{d}_j^R\ ; j=2,3,\dots,7$ can be obtained as follwos:

Iteration 1. Since only node 1 is the predecessor node of node 2 so putting

$i=1$ *and* $j=2$ in step 2 of the algorithm, the value of $\tilde{d}_2^R$ *is*

$$\tilde{d}_2^R=minimum\left(\tilde{d}_1^R+\tilde{d}_{12}^R\right)=minimum([(0,0,0):(0,0,0)]+[(9,12,15):(6,12,18)])=[(9,12,15):(6,12,18)]$$

Since minimum occurs corresponding to $i=1$, so label of node 2 as $\{[(9,12,15):(6,12,18)]\ ;1\}$
Iteration 2. since only node 2 is the predecessor node of node 3 so putting $i=2$ *and* $j=3$, the value of $\tilde{d}_3^R$ is

$$\tilde{d}_3^R=minimum\left(\tilde{d}_2^R+\tilde{d}_{23}^R\right)=\min([(9,12,15):(6,12,18)]+[(12,16,21):(7,16,25)])=[(21,28,36):(13,28,43)]$$

Since minimum occurs corresponding to $i=2$,
so label of node 3 as $\{[(21,28,36):(13,28,43)]\ ;2\}$
Iteration 3. the predecessor nodes of node 4 are node 1 and 3 so putting
$i=1,\ 3$ *and* $j=4$, the value of $\tilde{d}_4^R$ is

$$\tilde{d}_4^R=minimum\left(\tilde{d}_1^R+\tilde{d}_{14}^R\ ,\tilde{d}_3^R+\tilde{d}_{34}^R\right)=\min([(0,0,0):(0,0,0)]+[(20,25,30):(13,25,33)],[(21,28,36):(13,28,43)]+[(15,20,24):(12,20,27)])$$
$$=min([(20,25,30):(13,25,33)],\ [(36,48,60):(25,48,70)]$$

$$\Re[(20,25,30):(13,25,33)]=\frac{1}{8}(20+30+4(25)+13+33)=24.5$$

$$\Re[(36,48,60):(25,48,70)]=\frac{1}{8}(36+60+4(48)+25+70)=47.875$$

Since $\Re[(20,25,30):(13,25,33)]<\Re[(36,48,60):(25,48,70)]$ So
$minimum([(20,25,30):(13,25,33)],\ [(36,48,60):(25,48,70)])=[(20,25,30):(13,25,33)]$
Now we have $\tilde{d}_4^R=[(20,25,30):(13,25,33)]$
Since minimum occurs corresponding to $i=1$,
so label of node 4 as $\{[(20,25,30):(13,25,33)]\ ;1\}$
Iteration 4. the predecessor nodes of node 5 are node 2 and 3 , so putting
$i=2,3$ *and* $j=5$, the value of $\tilde{d}_5^R$ *is*

$$\tilde{d}_5^R=minimum\left(\tilde{d}_2^R+\tilde{d}_{25}^R\ ,\ \tilde{d}_3^R+\tilde{d}_{35}^R\right)=\min([(9,12,15):(6,12,18)]+[(6,11,16):(2,11,20)],[(21,28,36):(13,28,43)]+[(7,14,17):(5,14,19)])$$
$$=min([(15,23,31):(8,23,38)]\ ,\ [(28,42,53):(18,42,62)])$$

$$\begin{cases}\Re[(15,23,31):(8,23,38)]=\frac{1}{8}(15+31+4(23)+8+38)=23\\ \Re[(28,42,53):(18,42,62)]=\frac{1}{8}(28+53+4(42)+18+62)=41.125\end{cases}$$

So the $\tilde{d}_5^R=[(15,23,31):(8,23,38)]$
Since minimum occurs corresponding to $i=2$,
so label of node 5 as $\{[(15,23,31):(8,23,34)]\ ;2\}$
Iteration 5. the predecessor nodes of node 6 are node 4 and 5 , so putting
$i=4,5$ *and* $j=6$, the value of $\tilde{d}_6^R$ is

$$\tilde{d}_6^R=minimum\left(\tilde{d}_4^R+\tilde{d}_{46}^R\ ,\tilde{d}_5^R+\tilde{d}_{56}^R\right)=\min([(20,25,30):(13,25,33)]+[(20,25,28):(17,25,33)],[(15,23,31):(8,23,38)]+[(8,9,10):(5,9,13)])$$
$$=\min([(40,\ 50,\ 58):(30,\ 50,\ 66)]\ ,\ [(23,\ 32,\ 41):(13,\ 32,\ 51)])$$

$$\begin{cases}\Re[(40,50,58):(30,50,66)]=\frac{1}{8}(40+58+4(50)+30+66)=49.5\\ \Re[(23,32,41):(13,32,51)]=\frac{1}{8}(23+41+4(32)+13+51)=32\end{cases}$$

So the $\tilde{d}_6^R=[(23,\ 32,\ 41):(13,\ 32,\ 51)]$
Since minimum occurs corresponding to $i=5$,

so label of node 6 as
$\{ [(23,\ 32,\ 41):(13,\ 32,\ 51)] ; 5 \}$
Iteration 6. the predecessor nodes of node 7 are node 5 and 6 , so putting
$i = 5, 6\ and\ j = 7$, the value of $\tilde{d}_7^R$ is

$$\tilde{d}_7^R = minimum\left(\tilde{d}_5^R + \tilde{d}_{57}^R\ , \tilde{d}_6^R + \tilde{d}_{67}^R\right)$$

$$= min\begin{pmatrix} [(15,23,31):(8,23,38)] + \\ [(18,21,26):(15,21,31)]\ , \\ [(23,32,41):(13,32,51)] + \\ [(8,11,13):(4,11,14)] \end{pmatrix}$$

$$= \min([(33,44,57) : (23,44,69)]\ , [(31,43,54) : (17,43,65)])$$

$$\begin{cases} \Re[(33,44,57) : (23,44,69)] = \frac{1}{8}(33 + 57 + 4(44) + 23 + 69) \\ \quad = 44.75 \\ \Re[(31,43,54) : (17,43,65)] = \frac{1}{8}(31 + 54 + 4(43) + 17 + 65) \\ \quad = 42.375 \end{cases}$$

So the $\tilde{d}_7^R = [(31,43,54) : (17,43,65)]$
Since minimum occurs corresponding to $i = 6$,
so label of node 7 as $\{ [(31,43,54) : (17,43,65)] ; 6 \}$
Iteration 7. Since node 7 is the destination node of the given network, now the optimal shortest path between source node 1 and destination node 7 can be obtained by using the following procedure:
Since node 7 is labeled by
$\{ [(31,43,54) : (17,43,65)] ; 6 \}$ which represents that we are coming from node 6, check the label of node 6?.
Node 6 is labeled by
$\{ [(23,\ 32,\ 41):(13,\ 32,\ 51)] ; 5 \}$ which represents that we are coming from node 5, check the label of node 5?. Node 5 is labelled by $\{[(15,23,31):(8,23,34)\ ] ; 2 \}$ which represents that we are coming from node 2, check the label of node 2 ?. Node 2 is labeled by
$\{ [(9,12,15):(6,12,18)\ ] ; 1 \}$ which represents that we are coming from node 1. Now the optimal shortest path between node 1 and node 7 is obtained by joining all obtained nodes. Hence the optimal shortest path is $1 \to 2 \to 5 \to 6 \to 7$, with optimal fuzzy rough shortest distance between source node 1 and destination node 7 is
$\tilde{A}^R = [(31,43,54) : (17,43,65)]$.
The shortest path and the corresponding distention using algorithms are reported below:
optimal shortest path from node 1 to node 7 is: $1 \to 2 \to 5 \to 6 \to 7$
optimal fuzzy rough shortest distance from node 1to node 7 is :
$\tilde{A}^R = [(31,43,54) : (17,43,65)]$, and its membership function can be defined as:

$$\tilde{A}^R = \begin{cases} \mu_{\tilde{A}^L}(x) = \begin{cases} \dfrac{x-31}{12} & 31 \le x \le 43\ , \\ \dfrac{x-54}{-11} & 43 \le x \le 54\ , \\ 0 & otherwise \end{cases} \\ \mu_{\tilde{A}^U}(x) = \begin{cases} \dfrac{x-17}{26} & 17 \le x \le 43\ , \\ \dfrac{x-65}{-22} & 43 \le x \le 65\ , \\ 0 & otherwise \end{cases} \end{cases}$$

Also the membership function of $\tilde{A}^R$ is shown in Figure 3.

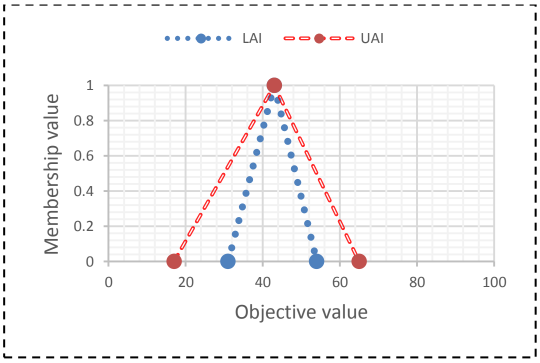

Figure 3. The membership of $\tilde{A}^R$

The fuzzy rough shortest distance and shortest path of all the nodes from node 1, and the labelling of each node is shown in Table 2.

Table 2. Representation of different shortest paths			
Node (j)	$\tilde{d}_j^R$	Shortest path between node 1^{st} and node j^{th}	The label of each node
2	$[(9,12,15):(6,12,18)]$	$1 \to 2$	$\{\ \tilde{d}_2^R - 1\ \}$
3	$[(21,28,36):(13,28,43)]$	$1 \to 2 \to 3$	$\{\ \tilde{d}_3^R - 2\ \}$
4	$[(20,25,30):(13,25,33)]$	$1 \to 4$	$\{\ \tilde{d}_4^R - 1\ \}$
5	$[(15,23,31):(8,23,38)]$	$1 \to 2 \to 5$	$\{\ \tilde{d}_5^R - 2\ \}$
6	$[(23,32,41):(13,32,51)]$	$1 \to 2 \to 5 \to 6$	$\{\ \tilde{d}_6^R - 5\ \}$
7	$[(31,43,54):(17,43,65)]$	$1 \to 2 \to 5 \to 6 \to 7$	$\{\ \tilde{d}_7^R - 6\ \}$

IV. CONCLUSION

We presented algorithm for computing a shortest path and fuzzy rough shortest distance between source node and destination node. Using the ranking function to compare fuzzy numbers in the algorithm. By using algorithm we can find out fuzzy rough distance and shortest path of each node from the source node simultaneously, i.e., it is not required to apply the algorithm again and again for finding fuzzy rough distance and shortest path for a particular node from source node see Table 2.

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