KALMAN FILTER Q-R EMPIRICAL OPTIMIZATION

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Article information	Abstract

Key words

Kalman Filter; Luenberger State Observer; Optimal State Estimation; Empirical Optimization; Stochastic Processes.

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In this paper, in order to simplify the application of the Kalman filter in practice, and as a continuation to the work presented in [14], the Kalman filter empirical optimization is redone from the design parameters point of view, which are the variances of the input disturbance Q and the output measurement error R, where, for a range of Q and R values, the optimal Kalman filter performance is determined empirically also for the specified process parameters and conditions. Furthermore, from the last step, a three dimensional graph of the optimal Kalman filter gain versus Q-R is constructed and presented as a normalized general solution to Kalman filter optimal innovation gain, also, the normalized graph can be put and used as general solution in table format.

I. Introduction

Kalman filter is an optimal stochastic state estimator, it is applied to estimate the states of a stochastic process with linear dynamic behavior that has normally distributed stochastic input and output disturbances and can be realized as discrete or continuous filter [1-13]. Usually, in control engineering, the stochastic input disturbance models the effect of a real disturbance or a modelling error (uncertainty) or both of them together, while the output stochastic disturbance models the random errors induced in the measurements. Typically, the input disturbance is a real external or internal (physical) action disturbing the process that needs to be regarded and compensated, while the output disturbance is considered as a stochastic measurement error, which is a fake not real value, that contaminates the true real output during the measurement process, this is also called measurement noise alternatively.

However, in order to design the Kalman filter to estimate the state(s) optimally, the input and the output disturbance parameters of the stochastic process are needed. In other words, to make the Kalman filter optimal, the variances of the input and the output disturbances of the stochastic process have to be known.

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In most of control engineering practice, the variance of the output disturbance can be computed from the available measurements, on the contrary to the input disturbance, the variance of the input disturbance is unkown or cannot be measured directly in most practical cases and has to be guessed or estimated. The availability of output disturbance variance only makes the design of the Kalman filter a trial and error design strategy.

Therefore, in this paper, the empirical optimization of the Kalman filter is done using the input and output variances as design parameters, this is as continuation to the previous work [14], where the empirical optimization of the Kalman filter is done from the point view of Kalman filter innovation gain. Also, this type of empirical optimization was applied on the Luenberger observer empirical optimizations in [15, 16 & 17].

The problem formulation of this paper is to execute a series of experiments with different parameters and conditions of the stochastic process, which are the input and output variances as well as the process dynamic parameters, and then a range of Kalman filters are designed and tested for a range of input and output disturbance variances Q-R, furthermore, the Sum of Squared Error (SSE) is computed between the true real state of the process and the estimated state for every single experiment.

The structure of the paper is as following, the description of the empirical Q-R optimization experimental setup is given in section II, then the first and the second serieses of experiments are given in sections III and IV, moreover, the generalized and normalized optimal Kalman gain versus Q-R graph is presented in section V. Finally, some conclusions are presented in section VI, and the list of references is given at the end.

II. Experimental Setup

Now, a series of experiments are conducted for specific conditions, in terms of, process parameters, particularly, the stochastic input and output disturbance parameters, specified by process input disturbance and output measurement noise variances. The mathematical model of the real stochastic process, which is to be measured (observed), is defined by the following equations

$$x(n+1) = ax(n) + b[u(n) + w(n)], \tag{1a}$$

$$y(n) = cx(n) + v(n), \tag{1b}$$

where w and v are normally distributed independent stochastic variables with variances Qt and Rt respectively, and a, b & c are the process dynamic parameters, u, x, y are the process known input, the state and the measured output respectively.

Then, a steady state Kalman filter is designed for a range of Q and R and the performance is measured empirically by computing the Sum of Squared Error (SSE) between the true system state of the stochastic process and the estimated Kalman state. For computing the steady state optimal Kalman gain, it is either to solve the Ricati equation or to compute it by using preiori state estimation variance propagation equation

$$P(n \backslash n-1) = aP(n-1 \backslash n-1)a + bQb, \tag{2}$$

the Kalman gain equation

$$K_n = \frac{P(n\backslash n-1)c}{cP(n\backslash n-1)c+R}\ , \tag{3}$$

and posteriori state estimation variance propagation equation

$$P(n\backslash n) = (1-K_n c)P(n\backslash n-1), \tag{4}$$

iteratively starting by an initial covariance estimate until it reaches the steady state, which is usually about after ten iterations, alternatively, the matlab function (kalman) can also be used to compute the steady state gain.

Then the resulted Kalman optimal gain is used in the steady state Kalman filter, see figure 1, to estimate the system states by the following equations

$$\hat{x}(n\backslash n-1) = a\hat{x}(n-1\backslash n-1) + bu(n-1), \tag{5}$$

$$\hat{x}(n\backslash n) = \hat{x}(n\backslash n-1) + K_{ss}[y(n) - c\hat{x}(n\backslash n-1)], \tag{6}$$

where K_{ss} is the computed steady state optimal Kalman filter gain. Finally, after collecting the data from running the series of experiments, and in order to assess the optimization empirically, all (Q, R) pairs and their corresponding SSE values are presented in three dimensional (3D) graphs.

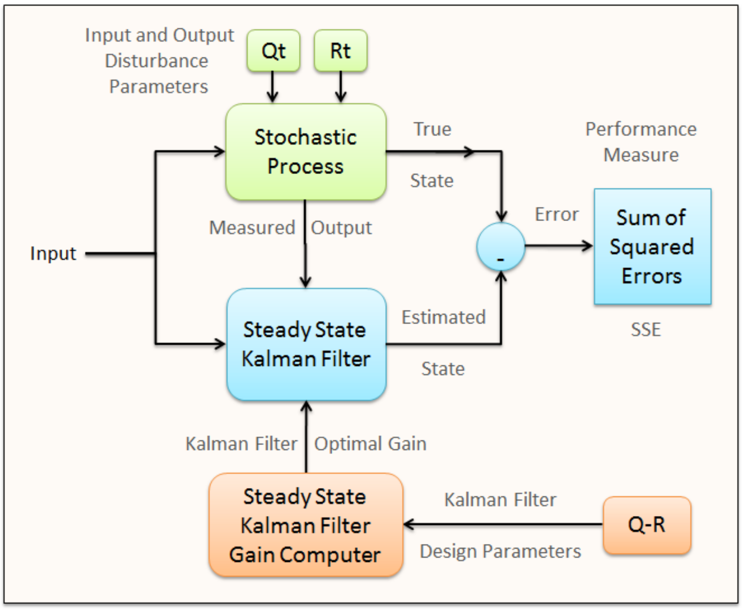

Figure 1: The block diagram of the Kalman filter Q-R empirical optimization tests.

III. First Series of Experiments

Here, the process true parameters are given by (a =0.5, b=1, c=1, Qt=0.01, Rt=0.01), and an empirical optimization test is run with a series of experiments with the range of Q and R (0 to 10), the SSE performance index is plotted against its corresponding Q-R pair in every single experiment. This results a 3D graph, which is shown in figure 2A.

The absolute minimum region is a line with slop of 1(Q=R). Figure 2B is the top view of the 3D graph presented in figure 2A, the figure shows clearly the optimal region is a straight line of the equation Q=R with a slop equal to 1, this is indicated in the graphs by extra dotted lines in figure 2 A and B.

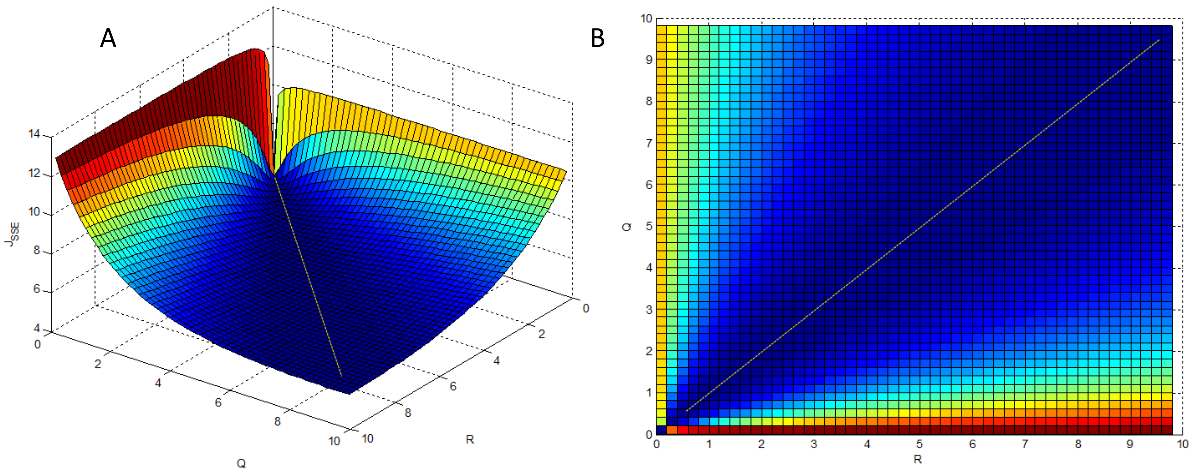

Figure 2: Empirical optimization for true parameters (Qt = 0.01, Rt = 0.01).

IV. Second Series of Experiments

In this case, the process true parameters are set up by (a =0.5, b=1, c=1, Qt=0.02, Rt=0.01). Again, the corresponding 3D graph is presented in figure 3A. Also its top view is plotted in figure 3B. By investigating the graphs, it is clear that the absolute minimum is a straight (dashed) line with a slop equal to 2, which means the optimal Kalman filter setup can be done by the relation Q=2R, for any value of R except zero.

From the results, It can be induced particularly for optimal tuning of Kalman filter in practice, it is not necessary to know the absolute values of Q and R but the relation between them. This has an attractive consistent practical appeal. Since in practice, we need only to assess the relative importance or the confidence between the model certainty and the measurement error variance to optimally tune the Kalman filter.

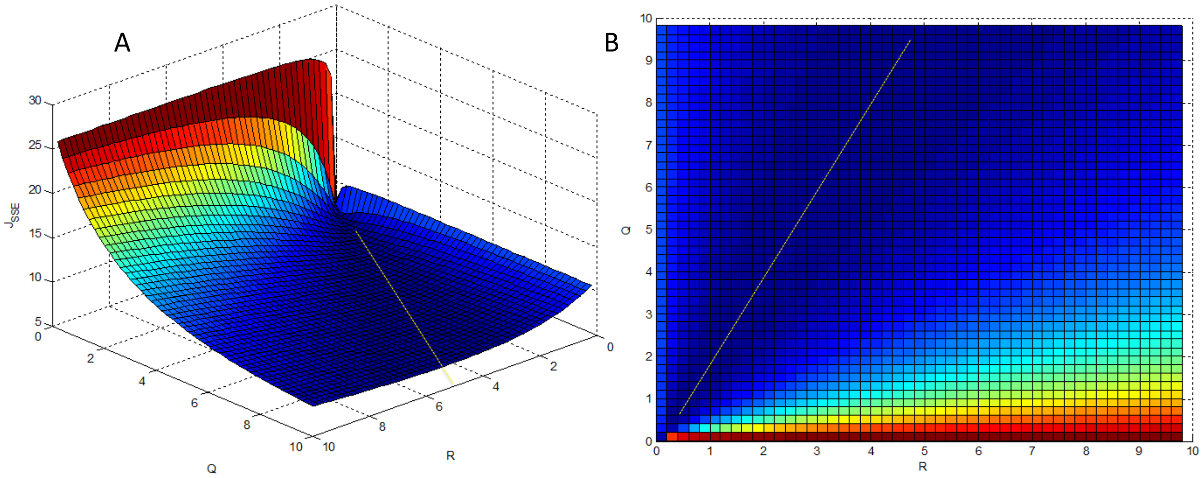

Figure 3: Empirical optimization for true parameters (Qt = 0.02, Rt = 0.01).

V. Kalman Filter Optimal Gain Versus Q-R Parameters

In this experiment, the steady state optimal Kalman gain is computed and plotted in three dimensional (3D) graph for a normalized range (0 to 1) of Q and R as shown in the figure 4A and its top view in figure 4B. Again the steady state Kalman gain is computed by iterating the Kalman variance propagaton equations, starting from an initial estimate variance until the Kalman gain reaches the steady state, as in the previous sections, which is about after ten iterations.

It is as expected earlier in previous section IV; the optimal Kalman gain depends on the ratio between variances of the input disturbance, or the model uncertainty, and the measurement error. This can be clearly seen from the 3D graph of figure 4A, that straight lines, starting from the origin parallel to Q-R plane, always have a constant value corresponds to the optimal Kalman gain. This can also be confirmed by looking at the top view of the same 3D graph shown in figure 4B. For example, if Q=0, this means that the process model is absolutely accurate or the process has no disturbances, then K_{man} gain is equal to zero, and here the filter will consider the model only and ignores the measurements for all R>0. On the other hand, if R=0, this means the measurements are absolutely accurate, then K_{man} gain is equal to one, the filter uses only the measurements and ignores the model for all Q>0. For other values Q and R not equal to zero, it is a straight line parallel to Q-R plane with slop equal to the ratio Q/R and height equal to the optimal (K_{man}) gain.

The resulted normalized optimization graph can be used to compute the optimal Kalman gain directly for given Q and R or their ratio Q/R instead of using the recursive algorithms.

Moreover, a data table can be generated with a desired resolution, so that the Kalman gain can be determined directly for any Q and R values or ratios with an interpolation algorithm.

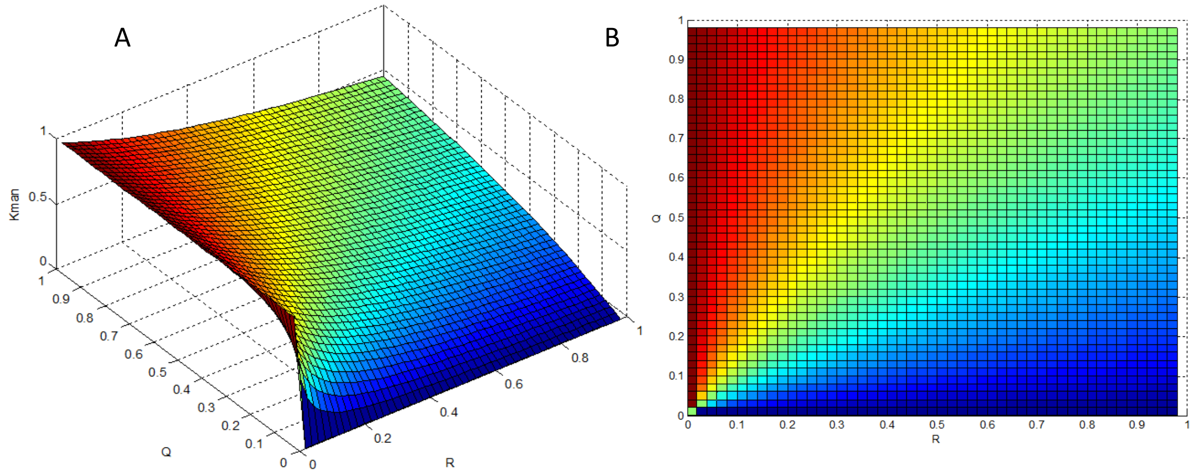

Figure 4: Normalized optimal Kalman gain versus Q and R.

VI. CONCLUSIONS

The optimal design of Kalman filter does not depend on the absolute values of modeling uncertainty or disturbance variance and the measurement error variance but rather on the ratio between them. This is a practical advantage since the tuning of the filter can be reduced down to one parameter instead of two, particularly, for example, when the modelling uncertainty or the disturbance variance is unknown, while the variance of the measurement is known.

Finally, the normalized three dimensional graph for the optimal Kalman gain versus the covariances of the modeling and the measurement errors can be used as precomputation for Kalman filter algorithm, where at any time, if the modeling error covariance and the measurement error covariance or the ratio of them are given then the Kalman gain can be determined directly from the normalized graph or its data table. This could help improve the performance of the filter in real time applications, particularly, if it is implemented within a computationally limited micro-controller based embedded system.

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التحسين التجريبي R-Q لمرشح كالمان

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الملخص

في هذه الورقة، ومن أجل تبسيط فهم مرشح كالمان وتطبيقاته العملية، وتكملة للعمل المنشور في البحث [14] ، تم إعادة أختبارات التحسين التجريبية لمرشح كالمان من وجهة نظر معاملات التصميم، والتي هي تباين إزعاج الدخل **Q** و تباين خطاء القياس **R**، حيث تم تحديد أداء مرشح كالمان الأمثل لمجموعة أو مجال من قيم **Q** و **R** تجريبياً بالإضافة لمعاملات وظروف النظام المحددة. بالإضافة الى الخطوة الأخيرة، تم إنشاء رسم بياني ثلاثي الأبعاد لكسب مرشح كالمان الأمثل مقابل **R-Q** وتقديمه كحل عام موحد لكسب الابتكار الأمثل لمرشح كالمان، مع امكانية وضع الرسم البياني الثلاثي الأبعاد في شكل جدول واستخدامه كحل عام.